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BOOK REVIEW

Relativistic Numerical Hydrodynamics. By JAMES R. WILSON & GRANT J. MATHEWS. Cambridge University Press, 2003. 232 pp. ISBN 0521 631556. £60.00.
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The laboratories at Los Alamos and Lawrence Livermore were created to solve scientific problems associated with nuclear and missile based weapons. The complexity of these problems required powerful numerical methods for their solution. At Los Alamos, the legendary Frank Harlow invented the PIC and MAC methods for multi-phase and free-surface problems. For other problems the methods used were not new, but they were put together in a way that made them more accurate and more efficient. The book under review comes into the latter category. The senior author James Wilson has worked at the Lawrence Livermore for at least 3 decades and during that time he has made important contributions to improving the efficiency and accuracy of finite difference codes for Special and General Relativity. His co-author Grant Mathews has also made many significant contributions in both theoretical and numerical relativity.

Research papers usually leave out many of the details of numerical computations that would save the reader much frustration if they were known. The usual dictum, that every paper should carry enough detail to enable the computation to be reproduced by some other scientist, is not always followed. For that reason the present volume is especially welcome, because it attempts to provide those missing details.

The first chapter is a useful summary of General Relativity (GR) and the issues involved in solving the GR equations numerically. In Chapter 2 the authors begin their discussion of numerical techniques by working out finite difference equations for Special Relativity. In many ways this chapter sets the tone for the rest of the book. Their algorithms are described in detail, combining both physical and mathematical arguments in an instructive manner. However, the approach is often idiosyncratic. Those who work with Riemann solver techniques, or those who use particle methods such as PIC or SPH, may be astonished to read that integrating the equation for the internal energy rather than the total energy per unit volume, or particle, is the best approach. Most of us have found that integrating the total energy per unit volume is effective and natural because, in relativity, momentum and energy go together.

The authors argue strongly for using artificial viscosity rather than the implicit dissipation of the Riemann or TVD methods, and they are able to obtain good results with their artificial viscosity. It must be remembered that a number of different possible viscous terms have been proposed for special relativity and all have been found to be unstable. The seductive argument that goes along the lines provided a term with the right invariance properties and the right non relativistic limit is introduced into the non dissipative Hamiltonian then physically sensible equations will follow, fails miserably. This reviewer found it perplexing therefore that the authors introduce viscosity via the equations originally proposed by Eckart. These viscous terms were shown by Lindblom and others to lead to instability. In the non-relativistic case the choice of artificial viscosity is guided by secure theory for gases that are not too dense, and by experiment. In the relativistic case the problem is more difficult because we need to include the effect of the relativistic fields due to the particles. A simple case is a relativistic plasma where the viscosity is due to the full electromagnetic fields which may not be in equilibrium with the matter. The end result is a collection of approximate models. The errors are unknown, or poorly estimated, except when the particle number density is very low.

Because their adopted viscosity results in momentum being transferred at speeds faster than light the authors make use of flux limiters. In other numerical methods with an artificial viscosity there is no acausal behaviour because nothing is moved more than a resolution length in any time step, and the time step is limited by a Courant condition using the speed of light as a signal speed. Of course once an artificial viscosity is used (with or without a flux limiter) the original energy momentum tensor has been left behind, and the construction is motivated by physical intuition. In the case of Riemann solvers a natural artificial viscosity appears in terms of changes across cell boundaries. For the application to heavy ion collisions it seems to this reviewer that the artificial viscosities are at best reasonable guesses justified in the end by whether or not they work.

Chapter 2 contains several tests of the authors algorithm. These include shock calculations for which the results are very satisfactory and may lead the reader to explore the use of an artificial viscosity for these and related problems. An interesting application is to heavy ion collisions that are currently of very great interest. The equation of state is too crude to compare with current physical models, but the calculation illustrates how to handle a 3-fluid system. It would have been useful to have had some discussion of other algorithms for this problem especially the PIC calculation of Harlow, Amsden, Fix and Strottman at Los Alamos.

Chapter 3 is concerned with fluid dynamics within the framework of GR. This chapter, like Chapter 2 is concerned with Wilsons approach. As a colleague remarked this chapter "reads like a handbook for Wilsons code". There are no references more recent than 1980, and the reader who wants a more comprehensive view of the subject should head for electronic reviews on the Living Reviews web site. Chapter 4 is concerned with applications to Cosmology. The reader who expects a discussion of modern cosmological problems including the formation of galaxies will be disappointed. Instead the authors concentrate on a brief discussion of metrics other than the FRW metric within the approximation of a planar model.

The phenomena of supernovae have been of special interest in the last several years because they have been used as Cosmological distance estimators. If you want to treat a supernovae as a standard candle you need to know its properties, and this means understanding how it is initiated, and how its radiation varies with time. Wilson has been involved in work on supernovae over a period of 25 years and this chapter gives a great deal of information about simulating them. Along the way we get practical information about flux limiters for neutrino transport, and a brief but useful discussion of convection. However, if you want to know about the work of others, for example Hillebrandts group at Munich who have pioneered the simulation of sub-grid burning, you will have to look elsewhere.

The book concludes with a short chapter on rotating stars and a longer chapter on the binary neutron stars. This latter problem has not yet been solved except by using approximations which fail just when the physics is getting interesting. Wilson and Mathews concentrate on an approximate model, the conformally flat model, which simplifies the full GR equations, but at the cost of including gravitational radiation only approximately. The reader who is interested in such models will find a brief guide to a numerical method with a reasonable list of references.

If your goal is to get some advice about the physical and numerical difficulties of solving the equations of relativistic hydrodynamics using methods similar to those used by Wilson and Mathews this book will be useful. If you want a comprehensive study of the available numerical methods then, sadly, you will be disappointed.

J. J. Monaghan